



**INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH  
TECHNOLOGY**

**LAPLACE TRANSFORM CALCULATION OF DARK SATURATION CURRENT IN  
SILICON SOLAR CELL INVOLVING EXCITONS EFFECTS**

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**ABSTRACT**

In this work, the resolution of the differential equation system governing the electrons and excitons densities in the base region of a silicon solar cell is derived, using the Laplace transform. To check the validity of this method, a comparative study is made between the present results for the saturation current density and those obtained by R. Corkish and al in similar conditions with the same cell parameters. Numerical simulations for a silicon solar cell in the dark without polarization allows to obtain the excess minority carriers and excitons densities, the saturation current density according to various parameters of the cell such as the depth of the base, the doping level, the binding coefficient between electron and exciton, the lifetime and the recombination velocity etc....

**Keywords:** Saturation current, Laplace transform, exciton, binding coefficient

**INTRODUCTION**

Most of equations describing physical phenomena are of differential type. The resolution of these equations requires mathematical methods with appropriate boundary and initial conditions. The excitons transport in solar cells is governed by complex differential equations of which the resolution is tedious [1], [2], [3]. The method used in this paper to find numerical solutions for electrons and excitons densities is based on Laplace transform. The numerical simulations applied in a silicon solar cell in dark without polarization make possible to obtain profiles of the saturation current density according to the various parameters of the cell.

**THEORY**

**Assumptions**

In this study, some assumptions are made:

- (1) Low-level injection is assumed.
- (2) The depletion approximation
- (3) Minority carriers in bulk regions are assumed to flow predominantly by diffusion.

(4) Within the depletion region the drift and diffusion currents are opposing and approximately equal in magnitude.

(5) Recombination in the depletion region is neglected.

**Theoretical calculation**

The continuity equations governing the electrons density and the excitons by taking into account the binding of carriers pairs into excitons are as follows [4]:

$$\frac{1}{q} \Delta J_e = U_{eh} - G_{eh} + B \quad (1)$$

$$\Delta \Phi = -U_x + G_x + B \quad (2)$$

For cell in the dark ( $G_{eh}=0$  and  $G_x=0$ )

The recombination terms can be simply expressed in terms of lifetimes:

$$U_{eh} = \frac{\Delta n}{\tau_e} \quad (3)$$

and

$$U_x = \frac{\Delta n}{\tau_x} \quad (4)$$

From the law of mass action, the binding rate is given by [5], [6]:

$$B = b(np - n^* n_x) \quad (5)$$

The differential equations are as follows:

$$D_e \frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{\tau_e} + b(\Delta n N_A - \Delta n_x \cdot n^*) \quad (6)$$

$$D_x \frac{d^2 \Delta n_x}{dx^2} = \frac{\Delta n_x}{\tau_x} - b(\Delta n N_A - \Delta n_x \cdot n^*) \quad (7)$$

Let us express the excitons density  $\Delta n_x$  according to the one for excess minority carriers in the base  $n$  starting from the equation (4).

$$\Delta n_x = \frac{1}{bn^*} \left( \frac{1}{\tau_e} + bN_A \right) \Delta n - \frac{D_e}{bn^*} \frac{d^2 \Delta n}{dx^2} \quad (8)$$

By replacing  $n_x$  in equation (7), the following differential equation is obtained:

$$\left[ \frac{D_e D_x}{bn^*} \right] \frac{d^4 \Delta n}{dx^4} - \left[ \frac{D_e}{bn^*} \left( bn^* + \frac{1}{\tau_x} \right) + \frac{D_x}{bn^*} \left( bN_A + \frac{1}{\tau_e} \right) \right] \frac{d^2 \Delta n}{dx^2} + \left[ \frac{1}{\tau_x} \left( \frac{N_A}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right] \Delta n = 0 \quad (9)$$

**Expression for excess minority carriers density**

In order to simplify our preceding differential equation, let us pose:

$$A_1 = \frac{D_e D_x}{bn^*} \quad (10)$$

$$A_2 = \left[ \frac{D_e}{bn^*} \left( bn^* + \frac{1}{\tau_x} \right) + \frac{D_x}{bn^*} \left( bN_A + \frac{1}{\tau_e} \right) \right] \quad (11)$$

$$A_3 = \left[ \frac{1}{\tau_x} \left( \frac{N_A}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right] \quad (12)$$

The differential equation (7) in simplified form becomes:

$$A_1 \frac{d^4 \Delta n}{dx^4} - A_2 \frac{d^2 \Delta n}{dx^2} + A_3 \Delta n = 0 \quad (13)$$

The Laplace transform of equation (13) gives:

$$\Delta n(P) = \frac{A_2 \sum_0^1 P^K \Delta n^{1-K} - A_1 \sum_0^3 P^{3-K} \Delta n^K}{A_1 P^4 - A_2 P^2 + A_3} \quad (14)$$

$\Delta n_o^k$  is the kth-derivatives of the density of the minority carriers in the base. Some of these values will be fixed and the others will be determined from boundary conditions [7].

$$\Delta n_{oe} = 10^{12} \text{ cm}^{-3}$$

The first derivative of the density of the excess minority carriers in the base is calculated starting from the boundary conditions [8].

$$\Delta n_{oe}^{(1)} = S_f \frac{\Delta n_{oe}}{D_e} \quad (15)$$

$$\Delta n_{oe}^{(2)} = 10^{20} \text{ cm}^{-5}$$

The 3<sup>rd</sup> derivative  $\Delta n_{oe}^{(3)}$  has no influence on the saturation current density profiles; it can be considered as zero.

The final expression obtained is equivalent to this:

$$\Delta n_{ep}(P) = \frac{-B_1 P^3 - B_2 P^2 + B_3 P + B_4}{A_1 P^4 - A_2 P^2 + A_3} \quad (16)$$

with

$$B_1 = A_1 \Delta n_{oe} \quad (17)$$

$$B_2 = A_1 \Delta n_{oe}^{(2)} \quad (18)$$

$$B_3 = A_2 \Delta n_{oe} - A_1 \Delta n_{oe}^{(2)} \quad (19)$$

$$B_4 = A_2 \Delta n_{oe}^{(1)} - A_1 \Delta n_{oe}^{(3)} \quad (20)$$

Let us determine the values  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6$  and  $\alpha_7$  so that:

$$\Delta n(P) = \frac{\alpha_1 P + \alpha_2}{\alpha_3 P^2 + \alpha_4} + \frac{\alpha_5 P + \alpha_6}{\alpha_3 P^2 + \alpha_7} \quad (21)$$

After calculation, we obtain differentials expressions of the constants  $\alpha_i$  (see appendix)

The expression for excess minority carriers density as a function of the base depth ( $0 \leq x \leq H$ ) is

$$\Delta n = \frac{\alpha_1}{\alpha_3} \cosh(\beta_1 x) + \frac{\alpha_2}{\alpha_3 \beta_1} \sinh(\beta_1 x) + \frac{\alpha_5}{\alpha_3} \cosh(\beta_2 x) + \frac{\alpha_6}{\alpha_3 \beta_2} \sinh(\beta_2 x) \quad (22)$$

**1.1. Expression of the excitons density**

The excitons density expression in the dark is deduced from equation (8):

After calculation, the following expression is obtained:

$$\Delta n_x = \left[ \frac{\alpha_1}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_1)^2 \alpha_1}{b n^* \alpha_3} \right] \cosh(\beta_1 x) + \left[ \frac{\alpha_2}{\alpha_3 \beta_1 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e \beta_1 \alpha_2}{b n^* \alpha_3} \right] \sinh(\beta_1 x) + \left[ \frac{\alpha_5}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_2)^2 \alpha_5}{b n^* \alpha_3} \right] \cosh(\beta_2 x) + \left[ \frac{\alpha_6}{\alpha_3 \beta_2 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e \beta_2 \alpha_6}{b n^* \alpha_3} \right] \sinh(\beta_2 x) \quad (23)$$

**Expression of the current density**

Assuming that electrons and excitons transports in the base are largely dominated by the diffusion, the expression of the current density according to the depth of the base, the binding coefficient and the doping level becomes as follows:

$$J = q D_e \frac{d}{dx} \Delta n + q D_x \frac{d}{dx} \Delta n_x \quad (24)$$

$$J(x) = \left[ q D_e \frac{\alpha_1 \beta_1}{\alpha_3} + q D_x \left[ \frac{\alpha_1 \beta_1}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_1)^3 \alpha_1}{b n^* \alpha_3} \right] \right] \sinh(\beta_1 x) + \left[ q D_e \frac{\alpha_2}{\alpha_3} + q D_x \left[ \frac{\alpha_2}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_1)^2 \alpha_2}{b n^* \alpha_3} \right] \right] \cosh(\beta_1 x) + \left[ q D_e \frac{\alpha_5 \beta_2}{\alpha_3} + q D_x \left[ \frac{\alpha_5 \beta_2}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_2)^3 \alpha_5}{b n^* \alpha_3} \right] \right] \sinh(\beta_2 x) + \left[ q D_e \frac{\alpha_6}{\alpha_3} + q D_x \left[ \frac{\alpha_6}{\alpha_3 b n^*} \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e (\beta_2)^2 \alpha_6}{b n^* \alpha_3} \right] \right] \cosh(\beta_2 x)$$

(25)  
 The expression of the saturation current density in the junction is obtained by considering  $x = 0$  in equation (25).

$$J_0 = q D_e \left( \frac{\alpha_2 + \alpha_6}{\alpha_3} \right) + q D_x \left\{ \left[ \frac{\alpha_2 + \alpha_6}{\alpha_3 b n^*} \right] \left( b N_A + \frac{1}{\tau_e} \right) - \frac{D_e}{b n^* \alpha_3} \left( (\beta_2)^2 \alpha_6 + (\beta_1)^2 \alpha_2 \right) \right\} \quad (26)$$

Comparisons will be made with the one given in [4].

$$J_0 = -\frac{1}{2} q D_e \left\{ \begin{matrix} \frac{1}{\epsilon_f^2} \left[ 1 + (M_\Delta + 2rM_{21}) \delta^{-\frac{1}{2}} \right] \\ + \epsilon_f^2 \left[ 1 - (M_\Delta + 2rM_{21}) \delta^{-\frac{1}{2}} \right] \end{matrix} \right\} n_{oe} \quad (27)$$

**Comparative study**

In fig. 1, the variation of the current density profile is represented with the binding coefficient.

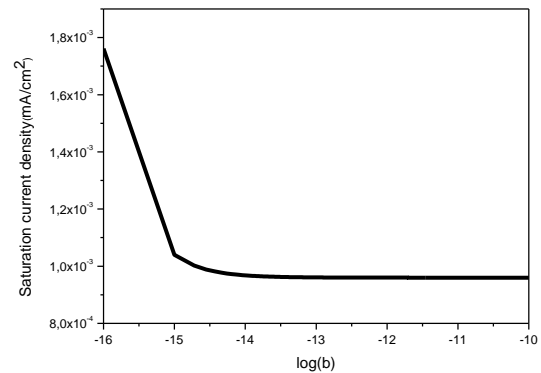


Fig.1. Saturation current density in function to the logarithm of the binding coefficient ( $N_A = 10^{18} \text{ cm}^{-3}$ ,  $H = 100 \text{ nm}$ ,  $S_f = 310^3 \text{ cm/s}$ ,  $\Delta n_{oe} = 10^{12} \text{ cm}^{-3}$ ,  $\Delta n_{oe}^{(2)} = 10^{20} \text{ cm}^{-5}$ ,  $D_e = 13 \text{ cm}^2/\text{s}$ ,  $D_x = 26 \text{ cm}^2/\text{s}$ )

The comparisons between this density profile and the one obtained from Corkish [4] in the same conditions and using the same parameters of the cell are displayed in figure(2):

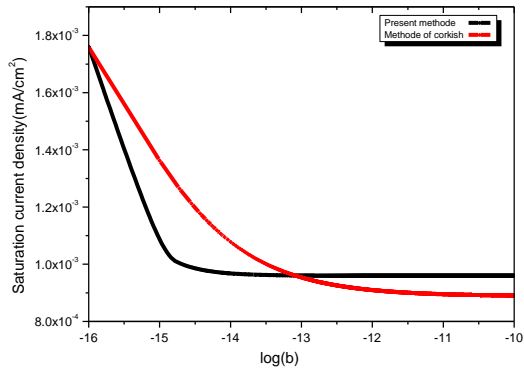


Fig. 2. Saturation current density in function to the logarithm of the binding coefficient ( $N_A = 10^{18} \text{ cm}^{-3}$ ,  $H = 100 \text{ nm}$ ,  $S_f = 310^3 \text{ cm/s}$ ,  $\Delta n_{oe} = 10^{12} \text{ cm}^{-3}$ ,  $\Delta n_{oe}^{(2)} = 10^{20} \text{ cm}^{-5}$ ,  $D_e = 13 \text{ cm}^2/\text{s}$ ,  $D_x = 26 \text{ cm}^2/\text{s}$ )

The comparison of the profiles shows a similar decrease of two the saturation current density according to the binding coefficient  $b$ .

An increase in this coefficient  $b$  causes strong volume recombination which reduces the density of the excess minority carriers in the base. Accordingly, a reduction in the current density is observed for these two profiles when the coupling becomes increasingly strong [9], [10].

The difference noted between the curves comes from the parameters  $\Delta n_{oe}^{(2)}$  and  $\Delta n_{oe}^{(3)}$  which respectively represent the second and third derivatives of the excess minority carriers the density at the junction given by the boundary conditions. This difference could be due to the fact that the derivatives of the density of the electrons to the junction should depend on the nature of the coupling.

### RESULTS AND DISCUSSION

From the present work, the current density can be examined as a function of binding coefficient through various cell parameters as doping level.

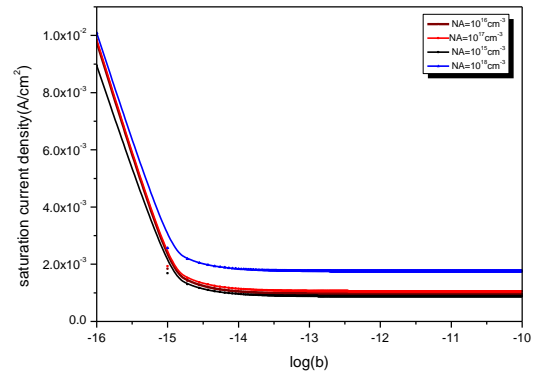


Fig. 3. Saturation current density in function to the logarithm of the binding coefficient for different doping level ( $H = 100 \text{ nm}$ ,  $S_f = 3.10^3 \text{ cm/s}$ ,  $\Delta n_{oe} = 10^{12} \text{ cm}^{-3}$ ,  $\Delta n_{oe}^{(2)} = 10^{20} \text{ cm}^{-5}$ ,  $D_e = 13 \text{ cm}^2/\text{s}$ ,  $D_x = 26 \text{ cm}^2/\text{s}$ )

The analysis of profiles density shows three parts with a first one corresponding to a very weak coupling ( $b = 10^{-15} \text{ cm}^3 \cdot \text{s}^{-1}$ ) or a very significant decrease of the current density with the binding coefficient. The second part for which  $10^{-15} \text{ cm}^3 \cdot \text{s}^{-1} \leq b \leq 10^{-13.6} \text{ cm}^3 \cdot \text{s}^{-1}$  is characterized by a weak variation of current density. The third part, corresponding to a strong coupling ( $b \geq 10^{-13.5} \text{ cm}^3 \cdot \text{s}^{-1}$ ), is characterized by a constant value of the saturation current density for each doping level. This is very comprehensible because for a strong coupling the rate of recombination reaches its maximum value. Thus, current density does not vary any more according to the binding coefficient making constant the excess minority carriers density and consequently that of the current. On the other hand, when the coupling is weak, the rate of recombination strongly increases and involves the reduction of the minority carriers density in the base [11].

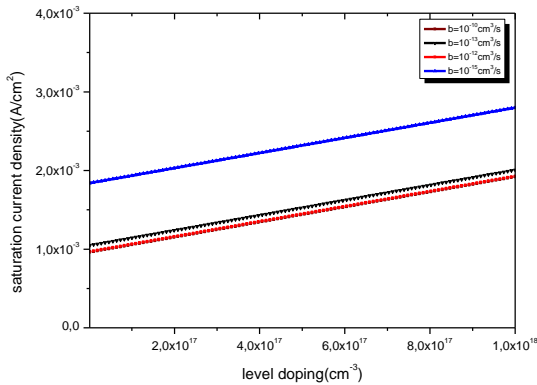


Fig. 4. Saturation current density in the base in function to the doping level for different values of the binding coefficient ( $H = 100\text{nm}$ ,  $Sf = 3.10^{-3} \text{ cm/s}$ ,  $\Delta n_{oe} = 10^{12} \text{ cm}^{-3}$ ,  $\Delta n_{oe}^{(2)} = 10^{20} \text{ cm}^{-5}$ ,  $D_e = 13 \text{ cm}^2/\text{s}$ ,  $D_x = 26 \text{ cm}^2/\text{s}$ )

Figure 4 shows a linear variation of the saturation current density according to the level doping for different values of binding coefficient. The same slope is obtained for any values of binding coefficient. It can be noted that when the coupling is strong ( $b > 10^{-13.4} \text{ cm}^3 \cdot \text{s}^{-1}$ ), the saturation current density profile is the same whatever the values of the binding coefficient in this interval, this confirms well the analyses made above because when the coupling is strong and the minority carriers density reaches its limit value. For weak coupling ( $b \leq 10^{-14} \text{ cm}^3 \cdot \text{s}^{-1}$ ), an increase of saturation current density is observed; this is simply due to a variation of the recombination rate at the junction. Indeed, the volume recombination caused by a strong coupling is reduced for small values of coefficient  $b$ . Consequently a significant increase in the density of the excess minority carriers implies the reduction of the saturation current density [12], [13].

### CONCLUSION

The use of Laplace transform in the resolution of the differential equations system governing the excess minority carriers and excitons density in the base of a silicon solar cell, allows having a better knowledge on the influence of excitons on the current density in the base. It is a very simple method of resolution compared to that used by R. Corkish and al. which

requires more thorough in algebra. This method enables to define two additional parameters in the base which are the second and third derivative of the density of the excess minority carriers to the junction. Thus for a good understand of these two parameters, the studies on their variation for different parameters of the solar cell and different boundary conditions could be carried out.

### Appendix

Determination of the  $\alpha_1$  coefficient from equations (16). The transformation of Laplace of the of the excess minority carriers in the base is given the following expression:

$$\Delta N_{ep}(P) = \frac{-B_1 P^3 - B_2 P^2 + B_3 P + B_4}{A_1 P^4 - A_2 P^2 + A_3}$$

Let us determine then expressions  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7$  such as

$$\Delta N_{ep}(P) = \frac{\alpha_1 P + \alpha_2}{\alpha_3 P^2 + \alpha_4} + \frac{\alpha_5 P + \alpha_6}{\alpha_3 P^2 + \alpha_7} \tag{A1}$$

We thus obtain the equality according to:

$$\frac{\alpha_1 P + \alpha_2}{\alpha_3 P^2 + \alpha_4} + \frac{\alpha_5 P + \alpha_6}{\alpha_3 P^2 + \alpha_7} = \frac{-B_1 P^3 - B_2 P^2 + B_3 P + B_4}{A_1 P^4 - A_2 P^2 + A_3} \tag{A2}$$

By identification we obtain the system of equation following:

$$\begin{cases} \alpha_3^2 = A_1 \\ \alpha_3 \alpha_7 + \alpha_3 \alpha_4 = -A_2 \\ \alpha_4 \alpha_7 = A_3 \\ \alpha_1 \alpha_3 + \alpha_3 \alpha_5 = -B_1 \\ \alpha_2 \alpha_3 + \alpha_3 \alpha_6 = -B_2 \\ \alpha_1 \alpha_7 + \alpha_5 \alpha_4 = B_3 \\ \alpha_2 \alpha_7 + \alpha_4 \alpha_6 = B_4 \end{cases} \tag{A3}$$

The resolution of this system gives the following expressions of  $\alpha_i$  according to the parameters of cell.

$$\alpha_3(b, N_A) = \sqrt{\frac{D_e D_x}{bn^*}} \tag{A4}$$

$$\alpha_4(b, N_A) = \frac{1}{2} \left[ \frac{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)}{\sqrt{\frac{D_e D_x}{bn^*}}} \right] + \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A5}$$

$$\alpha_7(b, N_A) = \frac{1}{2} \left[ \frac{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)}{\sqrt{\frac{D_e D_x}{bn^*}}} \right] - \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A6}$$

$$\alpha_6(b, N_A) = \frac{\frac{\Delta n_{oc} \left( \frac{D_e D_x}{bn^*} \right)^{\frac{3}{2}}}{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)} - \frac{1}{2} \Delta n_{oc} \sqrt{\frac{D_e D_x}{bn^*}}}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} + \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A7}$$

$$\alpha_2(b, N_A) = \frac{\frac{-\Delta n_{oc} \left( \frac{D_e D_x}{bn^*} \right)^{\frac{3}{2}}}{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)} + \frac{1}{2} \Delta n_{oc} \sqrt{\frac{D_e D_x}{bn^*}}}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} - \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A8}$$

$$\alpha_1(b, N_A) = \frac{\frac{\Delta n_{oc} \left( \frac{D_e D_x}{bn^*} \right)^{\frac{3}{2}}}{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)} - \frac{1}{2} \Delta n_{oc} \sqrt{\frac{D_e D_x}{bn^*}}}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} + \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A9}$$

$$\alpha_5(b, N_A) = \frac{\frac{\Delta n_{oc} \left( \frac{D_e D_x}{bn^*} \right)^{\frac{3}{2}}}{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)} - \frac{1}{2} \Delta n_{oc} \sqrt{\frac{D_e D_x}{bn^*}}}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} + \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A10}$$

$$\beta_1 = \sqrt{-\frac{\alpha_4}{\alpha_3}} \tag{A11}$$

And

$$\beta_2 = \sqrt{-\frac{\alpha_7}{\alpha_3}} \tag{A12}$$

we obtain the expressions following

$$\beta_2(N_A, b) = \frac{1}{2} \left[ \frac{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)}{\sqrt{\frac{D_e D_x}{bn^*}}} \right] - \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A13}$$

$$\beta_2(N_A, b) = \frac{1}{2} \left[ \frac{\frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)}{\sqrt{\frac{D_e D_x}{bn^*}}} \right] + \frac{1}{\sqrt{\frac{4 \frac{D_e D_x}{bn^*} \left[ \frac{1}{\tau_x} \left( \frac{1}{n^*} + \frac{1}{bn^* \tau_e} \right) + \frac{1}{\tau_e} \right]} + \frac{D_e}{bn^*} \left( \frac{1}{\tau_x} + \frac{D_x}{bn^*} \left( \frac{1}{bn^*} + \frac{1}{\tau_e} \right) \right)^2}} \tag{A14}$$

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